**Homework 7**

**P7.1.3** A parallel plate capacitor consists of two plates, each being  square, separated by 1 mm (Figure P7.1.3), as in the preceding problem. Half the space between the plates, in the horizontal direction, is filled by a dielectric of relative permittivity 10. Determine the capacitance, neglecting edge effects. Show that the capacitance can be considered as that of two capacitances in parallel.



**Solution:** The electric field *ξ* is the same throughout the space between the plates and is *V*/*d*. In the air-filled region *Da* = *ε*0*ξ* and is equal to the charge density on the plates. The charge on the plates in this region is *ε*0*ξAa,*where *Aa* = 5×2.5 cm2. In the dielectric-filled region, *Di* = *ε*0*εrξ* and is equal to the charge density on the plates. The charge on the plates in this region is *ε*0*εrξAi,*where *Ai* = 5×2.5 cm2. The total charge on the plates is *q* = *ε*0*ξ*(*Aa + εrAi*) = *ε*0*V*(*Aa + εrAi*)/*d.* The capacitance is  F = 121.7 pF.

The structure is equivalent to two capacitors in parallel, one filled with air, the other with dielectric. The capacitance of the air-filled capacitor is *ε*0*Aa*/*d* =  F ≡ 11.06 pF. The capacitance of the dielectric-filled capacitor is 10 times as much, or 110.63 pF. The equivalent parallel capacitance is (11.06 + 110.63) = 121.7 pF.

**P7.1.7** The current in a 1 μF capacitor is shown in Figure P7.1.7 as a function of time. Determine the total energy stored in the capacitor.



**Solution:** *q* at 4 ms is 10×4/2 = 20 μC. The energy is *w* = (20×10-6)2/(2×10-6) = 200 μJ.

**P7.1.11** The voltage applied to an initially-uncharged 0.1 μF capacitor is the first half-cycle of the waveform 10sin500*t* V, where  is in seconds. Derive the expressions, as functions of time, for the capacitor current, the energy stored in the capacitor, and the instantaneous power input to the capacitor. Sketch the time variation of these quantities.

**Solution:** *ω* = 500 rad/s, *T* = 2*π*/500 = 4*π* ms, and the duration of a half-cycle is 2*π* ms. *q*=*Cv*= 0.1×10-6×10sin(500*t*) C, where *t* is in s, or *q*= sin(0.5*t*) μC, ms, and *q* = 0 elsewhere.



*i*== 0.5cos(0.5*t*) mA, ms, and *i* = 0 elsewhere.

*w*== 5sin2(0.5*t*) μJ,ms, and *i* = 0 elsewhere.

*p*= 10sin(0.5*t*) ×0.5×cos(0.5*t*) = 5sin(0.5*t*)cos(0.5*t*) = 2.5sin(*t*) mW, and *p* =0, elsewhere.



**P7.1.15** The voltage waveform shown in Figure P7.1.15 is applied to a 1 μF capacitor. Determine the maximum value of the current through the capacitor.



Solution: ; Maximum current occurs during the rising phase of the voltage and equals: μA.

**P7.2.2** A series of voltage pulses of 10 mV amplitude and 2 ms duration are applied to an initially-uncharged inductor. How many pulses are required to bring the inductor current to

**Solution:** Flux linkage per pulse = 10×2 = 20 μVs; flux linkage for a current of 20 A is:  Vs. The number of required pulses is 5.

**P7.2.10** The triangular current pulse of Figure P7.2.10 is applied to aninitially-uncharged 0.1μH. Plot as a function of time: (a) the flux linkage in the inductor; (b) the energy stored in the inductor, (c) the instantaneous power input to the inductor.



**Solution:** *i* = A, s;



=A, s;

= 0, s.

(a)*λ* =μWb-turns, s;

=μWb-turns, s= 0, s.

(b)*w* = μJ, s;



=μJ, s;

= 0, s.

(c)*v* = μV, s;



=μv, s;

= 0, s, where μV may also beexpressed as 1 μWb-turns/min.



(d)*p* = μW, s;

= μW, s;

= 0, s, where μW may also be expressed as 10 μJ/min.

It is seen that . Thus μJ, s. At *t* = 60 s, *w* = 5 μJ. For s μJ.

**P7.2.13** The flux linkage in a 0.1 μH inductor varies with time as shown in Figure P7.2.13. Determine the average current through the inductor.



**Solution:** Average flux linkage is net area divided by the duration.The net area is Wb-Tmin. The average flux linkage is 6×10-6/(6) =

10-6Wb-T.the average current is 10-6/(0.1×10-6) = 10 A.



**P7.2.17** Determine *IX* in Figure P7.2.17.

**Solution:** Under dc conditions, the inductors behave as short circuits and the capacitors as open circuits. From current division, *IX* = (2/6)*ISRC* = *ISRC*/3 = 1 A.



**P7.2.19** The voltage *vL* shown in is applied to the initially uncharged inductor. Determine the value of *t* at which *iL* = -0.5 A.

**Solution:***iL* = -0.5 A when the flux linkage *λ* in the inductor reverses and is -0.5×1 = -0.5 Vs. At *t* = 3 s, the net positive area under the curve is: (1/2)2×2 – (1/2)2×1 = 1 Vs. An additional area, beyond 3 s, of -1.5is required to make the total area -0.5 Vs. This occurs at *t* = 3 *+* 0.5/2 = 3.75 s.